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A new model for radiation heat transfer in emitting, absorbing and anisotropically scattering media based on the concept of mean beam length

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Dedicated to 70th birthday of Professor Stephan

Abstract — A model to evaluate the total emissivity of an isothermal, gray, anisotropically scattering particle–gas mixture is illustrated. Based on the assumption, that the absorption path length differs from emission path length due to scattering, a mean beam length for the scattering medium is obtained. Comparison with exact results computed with a Monte Carlo technique showed good agreement in total emissivities when the mean beam length which is known from gas radiation is used. The influence of scattering albedo and scattering phase function is presented. © 2000 Éditions scientifiques et médicales Elsevier SAS

radiation / modelling / total emissivity / particle–gas mixture / mean beam length / scattering

Nomenclature

A	area	m^2
AMBL	absorption mean beam length defined by equation (9)	m^{-1}
d_p	particle diameter	m
i	intensity	$W \cdot m^{-2} \cdot sr^{-1}$
i_b	intensity of a blackbody	$W \cdot m^{-2} \cdot sr^{-1}$
K	factor introduced in equation (10)	
$\overline{K}$	constant value of K	
L	length	m
L_{mb}	mean beam length of a gas	m
L_{mb}^*	mean beam length of a scattering medium	m
m	complex refractive index	
p	scattering phase function	
Q	heat flow	W
R	radius of a sphere	m
s	length	m
T	temperature	K
X	particle size parameter	

Greek symbols

α	absorption coefficient	m^{-1}
γ	fit parameter defined by equation (16)	
ε	emissivity	
ε_∞	emissivity of an optical dense medium	
λ	wavelength	m
Θ	cone angle	rad
σ	scattering coefficient	m^{-1}
σ_B	Stefan–Boltzmann constant	$W \cdot m^{-2} \cdot K^{-4}$
ω	scattering albedo	
Ω	solid angle	sr

1. INTRODUCTION

The radiation heat flux leaving a gray gas depends on the temperature, the absorption coefficient of the gas α and the geometry. Under isothermal conditions the complexity due to different path lengths through the medium can be reduced by the concept of mean beam length L_{mb} [1]. L_{mb} is defined as the path length in an isothermal homogeneous gas which will result in an absorption of radiation equal to the absorption by the

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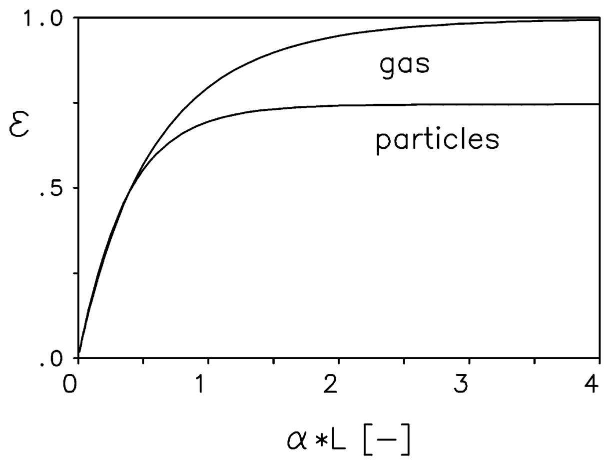

Figure 1. Total emissivities of an isothermal gas and an isothermal homogeneous cloud of particles as a function of optical density.

same gas inside the investigated geometry. With L_{mb} the total emissivities ε of technically important gases are shown in emissivity charts or can be calculated by the weighted sum of gray gases model [1].

For a gray gas the total emissivity can be determined by

$$\varepsilon = 1 - \exp(-\alpha \cdot L_{mb}) \quad (1)$$

For high optical thickness gray gases behave like a blackbody and the total emissivity approaches a limiting value of 1 (*figure 1*).

Replacing the gas by a homogeneous cloud of emitting, absorbing and scattering particles leads to considerable changes in emissivity. Corresponding to the scattering phase function the radiation path length from the place of emission to the bounding walls changes. While the energy which is emitted inside the medium is independant of scattering, the self absorption of the medium increases with increasing path lengths. Consequently there is less radiation leaving the geometry than from a nonscattering medium with the same absorption coefficient. As can be seen from *figure 1* the total emissivity approaches a limiting value < 1 with increasing optical thickness.

The scattering by the particles leads to a nonlinear propagation of thermal radiation.

In this paper a new model is developed to evaluate total emissivities of scattering isothermal media which is based on the concept of mean beam length.

2. GOVERNING EQUATIONS

2.1. Emissivity of an isothermal gas volume

The change of intensity due to absorption and emission of a homogeneous gray gas can be described by the equation of transfer (EOT)

$$\mathrm{d}i = -\alpha i \, \mathrm{d}s + \alpha i_b(T) \, \mathrm{d}s \quad (2)$$

with the absorption coefficient α of the gas and blackbody intensity i_b. Under isothermal conditions integration of equation (2) yields

$$i(s) = i(s=0)\exp(-\alpha s) + i_b\left[1 - \exp(-\alpha s)\right] \quad (3)$$

The heat flow Q, that is generated by the medium and reaches the bounding wall, can be found by integration of equation (3) over the bounding area A and all directions with boundary condition $i(s=0) = 0$

$$Q = \int_A \int_{2\pi} i_b\left[1 - \exp(-\alpha s)\right] \cos\Theta \, \mathrm{d}\Omega \, \mathrm{d}A \quad (4)$$

with cone angle Θ.

If Q is known the emissivity of the gas volume is given by equation (5)

$$\varepsilon = \frac{Q}{A \sigma_B T^4} \quad (5)$$

wherein σ_B is the Stefan–Boltzmann constant.

2.2. The concept of mean beam length L_{mb} for nonscattering media

The mean beam length L_{mb} which was originally introduced by Hottel [2] in the late twenties is defined as the constant path length that leads to the same heat flow Q as the considered geometry. With $L_{mb} = s = \mathrm{const}$ for a gray isothermal gas the integral in equation (4) can be solved with $i_b = \sigma_B T^4/\pi$ to give

$$Q = A \sigma_B T^4\left[1 - \exp(-\alpha L_{mb})\right] \quad (6)$$

Except of geometry the mean beam length depends on the optical density of the medium. L_{mb} decreases with increasing optical density. Nevertheless in practice usually a constant value for L_{mb} is used for calculations [3]. For various geometries these values can be taken from the literature, e.g. [3].

Rearranging of equation (6) according to equation (5) gives the total emissivity of the isothermal gas volume, equation (1)

$$\varepsilon = 1 - \exp(-\alpha \cdot L_{\mathrm{mb}})$$

2.3. The equation of transfer (EOT) for absorbing, emitting and scattering media

To describe the change of intensity in an isothermal emitting, absorbing and additionaly scattering medium like a homogeneous cloud of gray particles the EOT has to be extended by terms for scattering

$$\mathrm{d}i = -\alpha i \,\mathrm{d}s + \alpha i_{\mathrm{b}}(T)\,\mathrm{d}s - \sigma i \,\mathrm{d}s + \frac{\sigma}{4\pi}\int_{4\pi} i\left(\vec{s'}\right) p\left(\vec{s'} \to \vec{s}\right)\mathrm{d}\Omega'\,\mathrm{d}s \tag{7}$$

with the scattering coefficient σ. The scattering phase function $p(\vec{s'} \to \vec{s})$ is the relation of scattered intensity from $\vec{s'}$ direction in the $\vec{s}$ direction to the scattered intensity in the $\vec{s}$ direction if scattering were isotropic. From energy conservation the phase function is normalized as follows [3]:

$$\frac{1}{4\pi}\int_{4\pi} p\left(\vec{s'} \to \vec{s}\right)\mathrm{d}\Omega' = 1 \tag{8}$$

Because of the scattering from all directions, the EOT has to be solved simultaneously for all paths through the medium to gain the heat flow Q. In the following section, equation (7) is approximated by an equation that allows to disconnect all paths through the medium. This leads to a simplified model that describes the emissivity of isothermal scattering media with a mean beam length.

3. THE MEAN BEAM LENGTH FOR SCATTERING MEDIA

The emissivity of scattering media is frequently evaluated by use of the mean beam length known from gas radiation without any justification. Only a few papers are found dealing with the concept of mean beam length for scattering media.

Cartigny [4] describes an absorbing medium with isotropic single scattering by application of a probability model. The more the medium is optically thin or absorption dominates scattering ($\alpha \gg \sigma$) the more accurately the emissivity can be evaluated with the mean beam length known from gas radiation. This result is sound and agrees with the results from a Monte Carlo technique (see Section 4).

Yuen and Ma [5] suggested a new model. The heat flow Q from an isothermal homogeneous layer of gray particles between parallel plates is calculated with a zonal method. An "absorption mean beam length" *AMBL* is defined analogous to equation (6)

$$Q = A\sigma_{\mathrm{B}} T^4\left[1 - \exp(-\alpha \cdot AMBL)\right] \tag{9}$$

where *AMBL* depends on geometry and optical thickness as well as on scattering. A tabulation of "absorption mean beam lengths" had to consider not only different geometries, but also different scattering influences. Since the emissivity of a scattering medium approaches a limiting value < 1 with increasing optical thickness (*figure 1*), $\alpha \cdot AMBL$ approaches a finite limiting value with high optical density. Therefore the use of constant values for *AMBL* as practiced with L_{mb} for gas radiation is not possible, because the emissivity would approach $\varepsilon = 1$ with increasing absorption coefficient.

3.1. A simplified model for gray isothermal absorbing, emitting and scattering media

Due to the scattering the path lengths through the medium change. While the energy, which is emitted inside the medium is independant of scattering, the self absorption of the medium increases with increasing path lengths. As can be seen from the EOT for a gray gas (equation (2)), the path lengths of absorption and emission are the same for nonscattering media. Scattering will extend only the absorption path lengths by a factor K, while the emission path lengths remain unchanged. This leads to a simplified EOT for gray scattering media

$$\mathrm{d}i = -\alpha i K \,\mathrm{d}s + \alpha i_{\mathrm{b}}\,\mathrm{d}s \tag{10}$$

Comparison with the known EOT for scattering media (equation (7)) shows that K depends on α, σ and the scattering phase function $p(\vec{s'} \to \vec{s})$ as well as on the intensity distribution in the medium. Therefore K depends on the local position inside the medium.

Equation (10) can be integrated if K is assumed to be constant $K = \mathrm{const} = \overline{K}$ and the medium to be isothermal

$$i(s) = i(s=0)\exp\left(-\alpha\overline{K}s\right) + \frac{1}{\overline{K}}\, i_{\mathrm{b}}\left[1 - \exp\left(-\alpha\overline{K}s\right)\right] \tag{11}$$

By stating $K = \text{const}$ the intensity of the investigated path is disconnected from all other paths through the medium. This is no loss of accuracy, because for every single path through the medium a certain $\overline{K}$ can be found that leads to the correct intensity $i(s)$.

If $\overline{K}$ is assumed to be the same for all paths through the medium the heat flow Q that is generated by the medium and reaches the bounding wall can be found by integration of equation (11) over the bounding area A and all directions with boundary condition $i(s = 0) = 0$

$$Q = \frac{1}{\overline{K}} \int_A \int_{2\pi} i_b \left[1 - \exp\left(-\alpha \overline{K} s\right)\right] \mathrm{d}\Omega \cos\Theta \, \mathrm{d}A \quad (12)$$

A mean beam length for scattering media L^*_{mb} is defined analogous to L_{mb} as the constant path length that leads to the same heat flow as the given geometry

$$Q \equiv A\pi i_b \frac{1}{\overline{K}} \left[1 - \exp\left(-\alpha \overline{K} L^*_{mb}\right)\right] \quad (13)$$

and the emissivity ε of the isothemal scattering medium is

$$\varepsilon = \frac{1}{\overline{K}} \left[1 - \exp\left(-\alpha \overline{K} L^*_{mb}\right)\right] \quad (14)$$

As mentioned above the factor K is not a constant. In Section 5 will be shown that the assumption of $K = \overline{K}$ leads to good results of the calculated emissivities. A detailed discussion of K and $\overline{K}$ on the basis of a two-flux-model will be published [6].

The limiting value of ε with increasing optical thickness is from equation (14)

$$\varepsilon_\infty = \frac{1}{\overline{K}} \quad (15)$$

For $\overline{K} = 1$ equation (14) equals to equation (1). In this case the medium is nonscattering and L^*_{mb} equals L_{mb}.

To verify that the new model (equation (10)) leads to reasonable results of the emissivities when the factor K is treated as a constant, the heat flow Q is determined using a Monte Carlo technique. Different isothermal geometries are investigated with different values for α, σ and the scattering phase function $p(\vec{s'} \rightarrow \vec{s})$. The emissivity can be evaluated from equation (5). The factor $\overline{K}$ is calculated as the reciprocal of the emissivity of an optical thick medium (equation (15)). Solving equation (14) for L^*_{mb} using a numerical standard routine (Newton iteration) the results can be compared to the mean beam length L_{mb} known from gas radiation. This will be treated in Section 5.

In the following section the influence of scattering albedo $\omega = \sigma/(\alpha + \sigma)$, phase function and geometry on the emissivity of gray isothermal media will be shown. The heat flow Q is calculated with a Monte Carlo technique and equation (5) is solved to obtain the desired total emissivities.

4. THE EMISSION OF SCATTERING MEDIA

Figure 2 shows the emissivity of an isothermal gray spherical medium as a function of optical density for various values of the scattering albedo ω. The medium is supposed to scatter isotropically. The scattering albedo of particulate media in coal fired furnaces tends to lie within a range of $0 < \omega < 0.5$ when scattering is assumed isotropic, so the value of $\omega = 0.9$ is presented only to show the influence of scattering more clearly.

For small optical densities there is no significant influence of scattering on the emissivity. This is because nearly all emitted radiation reaches the boundary without being absorbed. With increasing optical density the emissivity approaches a limiting value of 1 (blackbody radiation) for nonscattering media and $\varepsilon < 1$ for scattering media. The limiting value decreases with increasing scattering albedo ω. With increasing influence of the scattering the path lengths from the place of emission inside the medium to the boundary increase due to the change of direction. This leads to an extended self absorption while the total emitted energy remains constant. Consequently, there is less radiation leaving the medium than from nonscattering media.

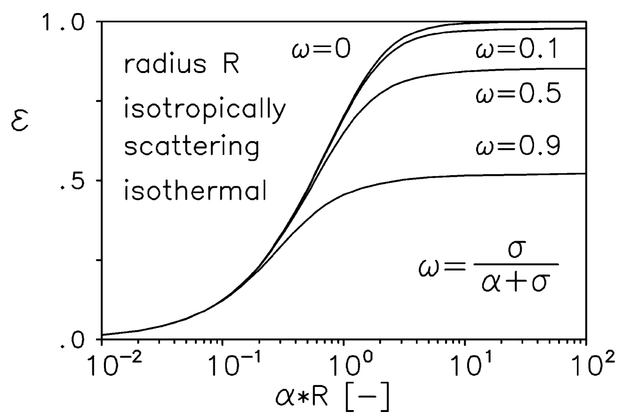

Figure 2. Total emissivity of an isothermal, isotropically scattering radiating sphere as a function of optical density and the scattering albedo ω.

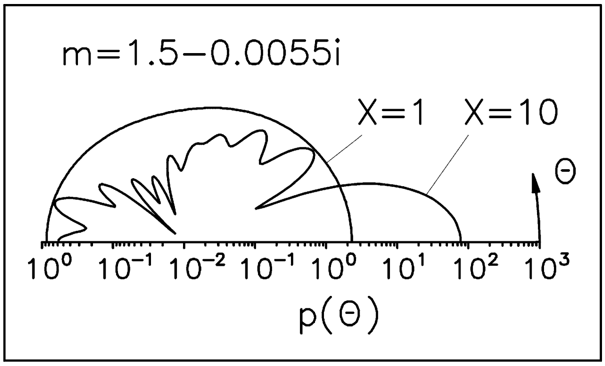

Figure 3. Scattering phase function as a function of scattering angle and particle size parameter X.

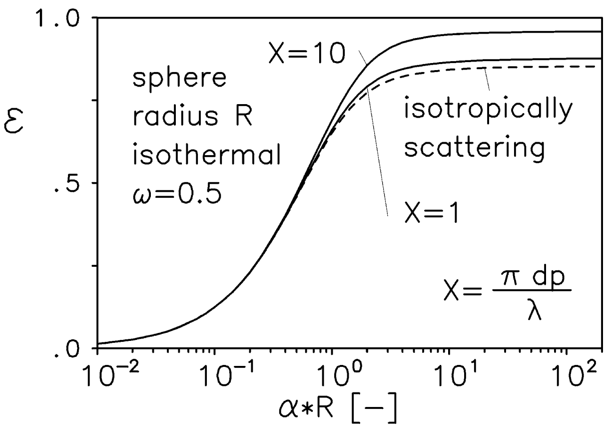

Figure 4. Total emissivity of an isothermal, isotropically scattering radiating sphere as a function of optical density and the scattering phase function.

To show the influence of the scattering phase function on the emissivity, the phase function $p(\vec{s}' \to \vec{s})$ was calculated using Mie theory for spherical particles for different values of the particle size parameter $X = \pi d_\text{p}/\lambda$. Mie theory is extensively described at Bohren and Huffman [7] where as well a FORTRAN routine for numerical calculations is printed. The particle cloud was supposed to be homogeneous with all particles of the same size. The complex refractive index was stated constant with $m = 1.5 - 0.0055i$, a representative value for the ashes of coal fired furnaces [8].

As can be seen from *figure 3* the larger particle size parameter X leads to a strong scattering in the forward direction while the particles with $X = 1$ scatter almost isotropically. Especially larger scattering angles tend to increase the path lengths from the point of emission to the boundary, so the smaller particles ($X = 1$) should influence the emissivity more drastically than larger

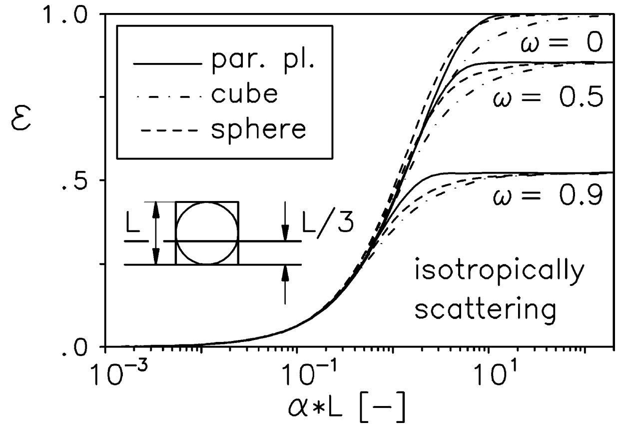

Figure 5. Total emissivities of different isothermal, isotropically scattering radiating geometries as a function of optical density.

particles. This result can be seen from *figure 4*, where the emissivity of an isothermal gray spherical medium is shown as a function of optical density. The scattering albedo was given as $\omega = 0.5$. The dashed line is the result of isotropic scattering and is printed for comparison. Forward scattering leads to an increase in emissivity compared to isotropic scattering.

When the emissivity is known, for calculating the mean beam length of a scattering medium L^*_mb from equation (14) the factor $\overline{K}$ is needed, too. From equation (15) $\overline{K}$ is the reciprocal value of the emissivity of an optical thick medium ε_∞ with the same scattering parameters as the investigated medium.

See the influence of geometry on ε_∞ first. *Figure 5* shows the emissivity from different geometries (a layer between infinite parallel plates, cube, sphere) as a function of optical thickness and with the scattering albedo ω as parameter. The scattering is isotropic. With increased optical density only a small region of the medium next to the bounding walls has significant influence on the emission. Therefore ε_∞ is the same for all geometries.

Using a Monte Carlo technique ε_∞ is determined and printed in *figure 6* as a function of the scattering albedo ω. The figure shows the results for isotropic scattering and for a phase function with strong scattering in the forward direction with a particle size parameter of $X = 10$ and a constant complex refractive index of $m = 1.5 - 0.0055i$ (*figure 3*).

A nonscattering medium with $\omega = 0$ has a limiting value of $\varepsilon_\infty = 1$. With increasing ω the emissivity of the optical dense medium decreases approaching $\varepsilon_\infty = 0$ for a non-emitting medium with $\omega = 1$. With decreasing scattering angles the emissivity increases as stated above.

The values for ε_∞ can be read from *figure 6*. For computational use a numerical approximation was found to be:

$$\varepsilon_\infty = \left(\frac{1-\omega}{1-\gamma\omega}\right)^{0.38} \quad (16)$$

The influence of the scattering phase function is taken account within the parameter γ. For isotropic scattering $\gamma \approx 0.5$ was calculated. For a homogeneous cloud of monodisperse particles with a complex refractive index of $m = 1.5 - 0.0055i$ the parameter γ is shown in *figure 7* as a function of the product of particle diameter and temperature.

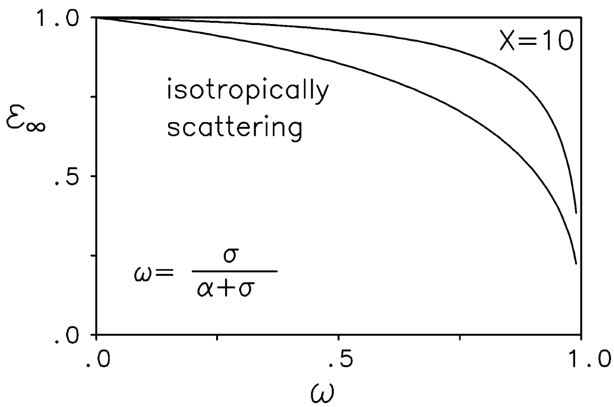

Figure 6. Total emissivity of an isothermal optical dense geometry as a function of the scattering albedo ω and the scattering phase function.

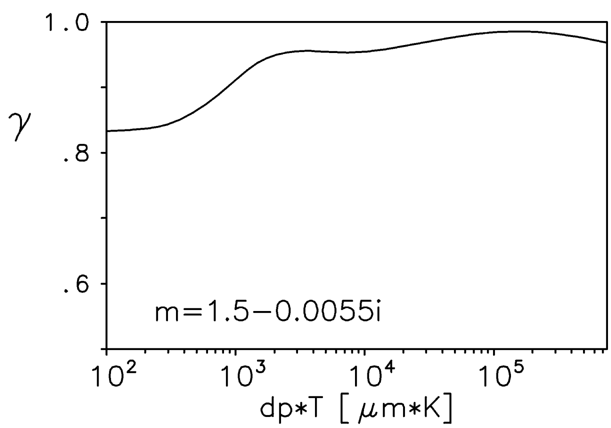

Figure 7. The parameter γ (defined by equation (16)) of a homogeneous cloud of particles with diameter d_p and a constant refractive index as a function of diameter and temperature.

5. THE MEAN BEAM LENGTH L^*_{mb} OF SCATTERING MEDIA

With ε and ε_∞ given, the corresponding value for L^*_{mb} of an isothermal scattering medium can be evaluated from equations (14) and (15) as stated above.

The normalized mean beam lengths L^*_{mb} of a sphere with radius R and a quad are shown in *figures 8* and *9*, respectively. The sets of curves are for various values of the scattering albedo ω. The scattering is assumed as isotropic. The treatment of anisotropic scattering will be presented in *figure 10*.

The curve for $\omega = 0$ shows the limiting case of a nonscattering medium, so L^*_{mb} corresponds to the known mean beam length from gas radiation L_{mb}. The other curves show L^*_{mb} for $\omega = 0.5$ and $\omega = 0.9$, respectively.

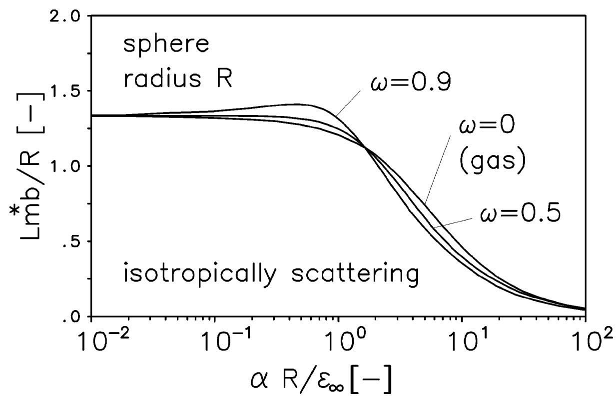

Figure 8. Dimensionless mean beam length of an isothermal sphere as a function of weighted optical density and scattering albedo. The scattering is assumed as isotropic.

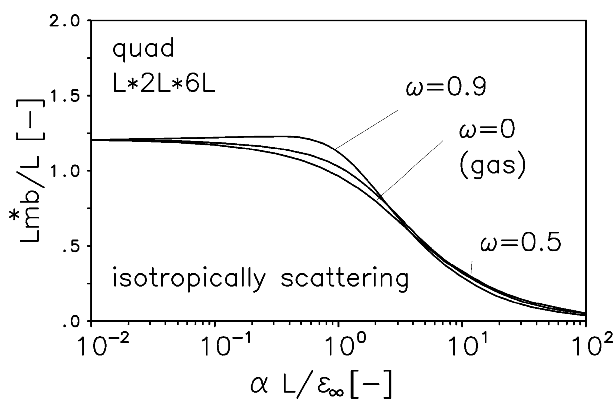

Figure 9. Dimensionless mean beam length of an isothermal quad (sidelengths $L \cdot 2L \cdot 6L$) as a function of weighted optical density and scattering albedo. The scattering is assumed as isotropic.

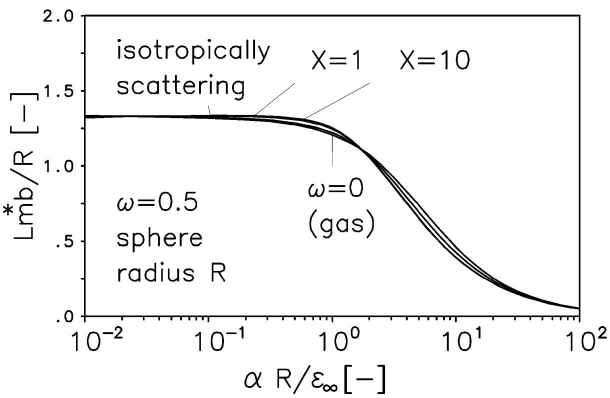

Figure 10. Dimensionless mean beam length of an isothermal sphere as a function of weighted optical density and scattering phase function.

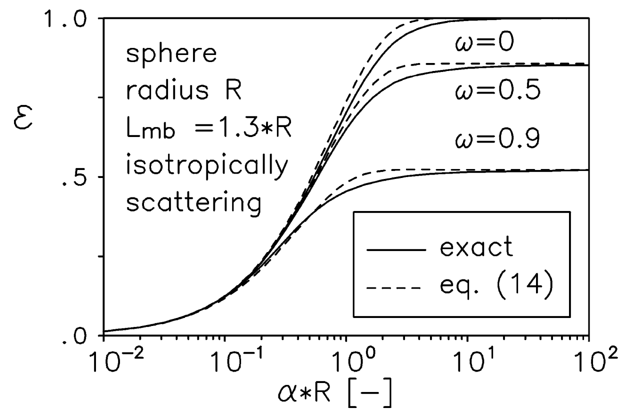

Figure 11. Total emissivity of an isothermal, isotropically scattering radiating sphere as a function of optical density and scattering albedo, calculated with a Monte Carlo technique and with the new model (dashed lines).

Even for $\omega = 0.9$, L_{mb} is a good approximation for L^*_{mb} of the scattering medium.

The small differences between L^*_{mb} and L_{mb} are the consequence of the assumption, that the factor K is constant within the medium. However, $K = \mathrm{const} = \overline{K}$ leads to a good match between L^*_{mb} and L_{mb}.

As could be expected from Section 4, when scattering is strongly forward directed, the difference between L^*_{mb} and L_{mb} decreases. This can be seen from *figure 10*, where the normalized mean beam lengths of a sphere with radius R for various scattering phase functions are shown. For two different particle size parameters, $X = 1$ and $X = 10$ and a constant complex refractive index $m = 1.5 - 0.0055i$ the scattering phase function was calculated with the Mie theory (see *figure 3*). The scattering albedo is $\omega = 0.5$. For comparison the curves for isotropic scattering and a nonscattering medium are shown as well. The difference between L^*_{mb} and L_{mb} decreases with increasing forward scattering.

Further geometries (layer between parallel plates, cube, cylinder) as well as the emission on parts of the boundary have been investigated. All cases prooved the mean beam length of a nonscattering medium L_{mb} to be a good approximation for L^*_{mb}.

To check the results the emissivity of an isothermal gray spherical medium with isotropic scattering is evaluated using the developed model (equation (14)). The factor $\overline{K}$ is taken from equation (15) as the reciprocal value of the emissivity of an optical dense medium, which can be calculated from equation (16) setting $\gamma = 0.5$ for isotropic scattering. The mean beam length L^*_{mb} is set equal to L_{mb} of a radiating gas and is assumed independant of optical density. As stated in Section 2.2 for engineering purposes usually a constant value for L_{mb} is applied. For a spherical medium with radius R a value of $L_{\mathrm{mb}} = 1.3 \cdot R$ can be found in the literature [3].

Figure 11 shows the emissivity from equation (14) as a function of optical thickness as dashed lines for various values of the scattering albedo ω. For comparison the exact results which were obtained by Monte Carlo calculations are given. For low and high optical density the new model agrees very well with the exact results. The differences for moderate optical thickness are partly due to the assumption that L_{mb} is a constant. This can be seen from the curves for $\omega = 0$. The calculation of ε with $L_{\mathrm{mb}} = L_{\mathrm{mb}}(\alpha R)$ leads to increasing accuracy of the new model.

6. EMISSION FROM A PARTICLE-GAS MIXTURE

The emissivity of a gas particle mixture can be determined with equation (14) using the absorption coefficient $\alpha_{\mathrm{gas+particles}}$ of the mixture. This is the sum of α_{gas} and $\alpha_{\mathrm{particles}}$. This leads to the following equation for the scattering albedo:

$$\omega = \frac{\sigma}{\alpha_{\mathrm{gas}} + \alpha_{\mathrm{particles}} + \sigma} \tag{17}$$

With ω from equation (17) ε_∞ is given from *figure 6* or can be calculated from equation (16) for numerical use.

Because the scattering albedo ω is evaluated with the absorption coefficient of the mixture, the scattering of radiation emitted from the gaseous phase is considered in the same way as the scattering of radiation emitted from the particulate phase.

7. CONCLUSIONS

Based on the assumption, that absorption path lengths differ from emission path lengths with a factor K, the exact equation of transfer for emitting, absorbing and scattering media (equation (7)) was replaced by equation (10). The influence of scattering on the intensity in a certain direction is expressed by K which is locally dependant.

With the assumption that $K = \overline{K}$ is a constant and under isothermal conditions, the resulting equation can be integrated. Analogous to gas radiation an equation for the emissivity of the scattering medium is obtained as a function of the factor $\overline{K}$ and a mean beam length:

$$\varepsilon = \frac{1}{\overline{K}}\left[1 - \exp\left(-\alpha \overline{K} L^*_{\text{mb}}\right)\right] \tag{18}$$

with $\overline{K} = 1/\varepsilon_\infty$.

$\overline{K}$ depends on the scattering albedo and the scattering phase function and is independant of the geometry of the medium.

Exact calculations with a Monte Carlo technique showed good agreement between L^*_{mb} and the mean beam length known from gas radiation.

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